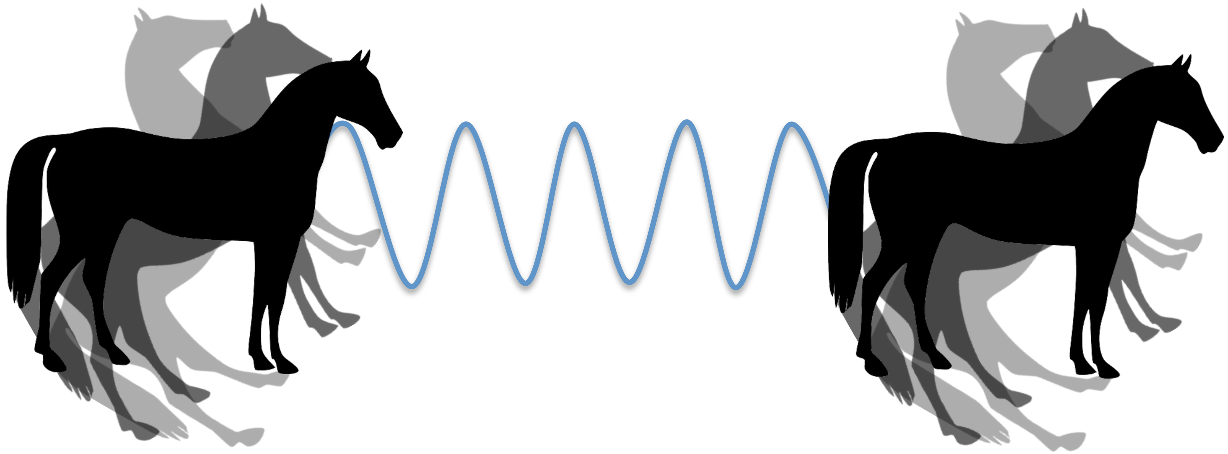
Einstein-Podolsky-Rosen steering provides the advantage in entanglement-assisted subchannel discrimination with one-way measurements

Marco Piani

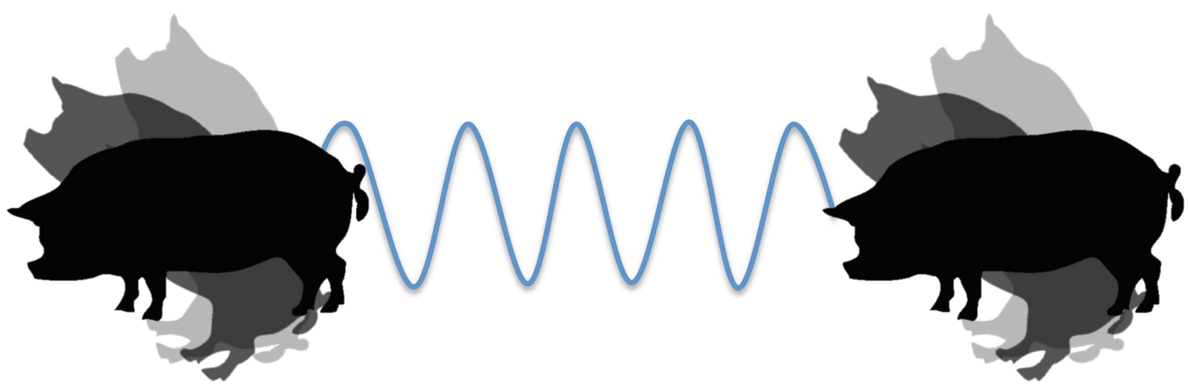
Joint work with John Watrous, arXiv:1406.0530, PRL to appear

QIP 2015, Sydney



“All entangled states are special,
but some are more special than others”

George Qrwell, *Entanglement farm*

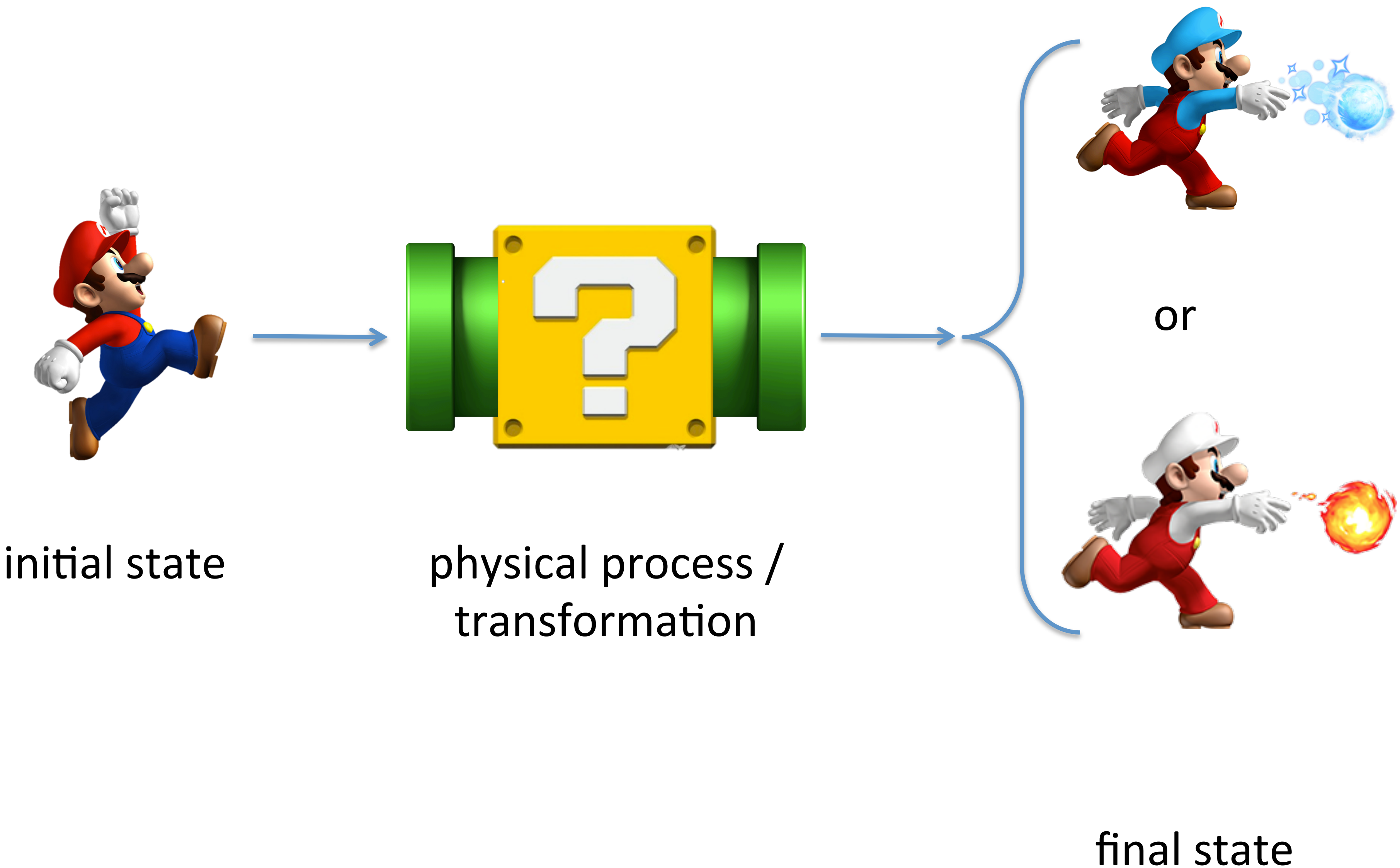


Goals:

- To understand quantum correlations
- To facilitate their exploitation

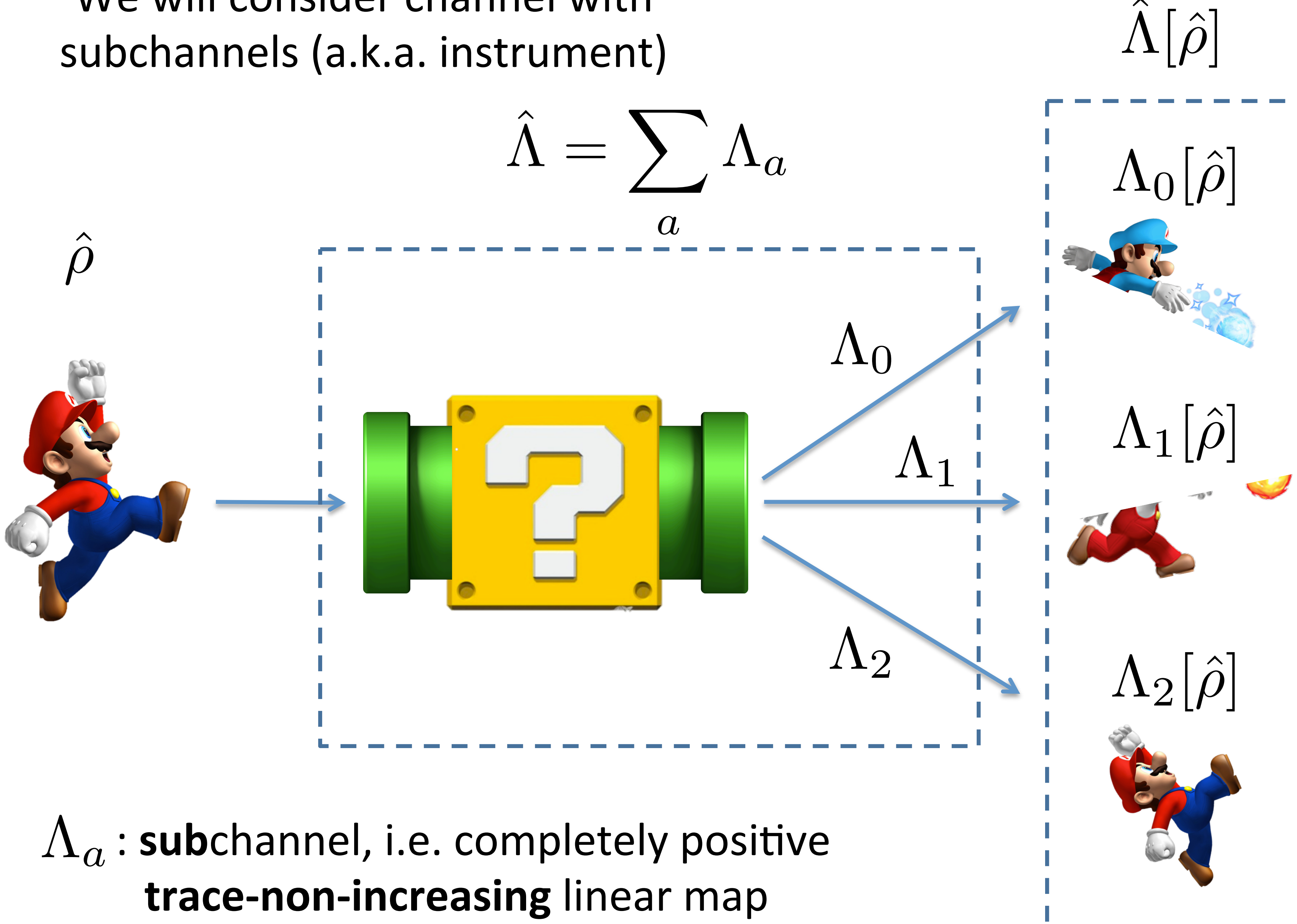
How:

Operational characterization
considering their usefulness in the discrimination
of physical processes


initial state
physical process / transformation
or
final state

We will consider channel with subchannels (a.k.a. instrument)

$$\hat{\Lambda} = \sum_a \Lambda_a$$



Λ_a : **sub**channel, i.e. completely positive **trace-non-increasing** linear map

Includes standard channel discrimination

$$\hat{\Lambda} = \sum_a \Lambda_a \qquad \Lambda_a = p_a \hat{\Lambda}_a$$

E.g.: $$\hat{\Lambda} = \frac{1}{2}\hat{\Lambda}_0 + \frac{1}{2}\hat{\Lambda}_1$$

but is more general...

EXAMPLE:

“Branches” of the amplitude damping channel

$$\hat{\Lambda} = \Lambda_0 + \Lambda_1$$

$$\Lambda_i[\hat{\rho}] = K_i \hat{\rho} K_i^\dagger$$

$$K_0 = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$$

$$K_1 = \sqrt{\gamma}|0\rangle\langle 1|$$

Task:

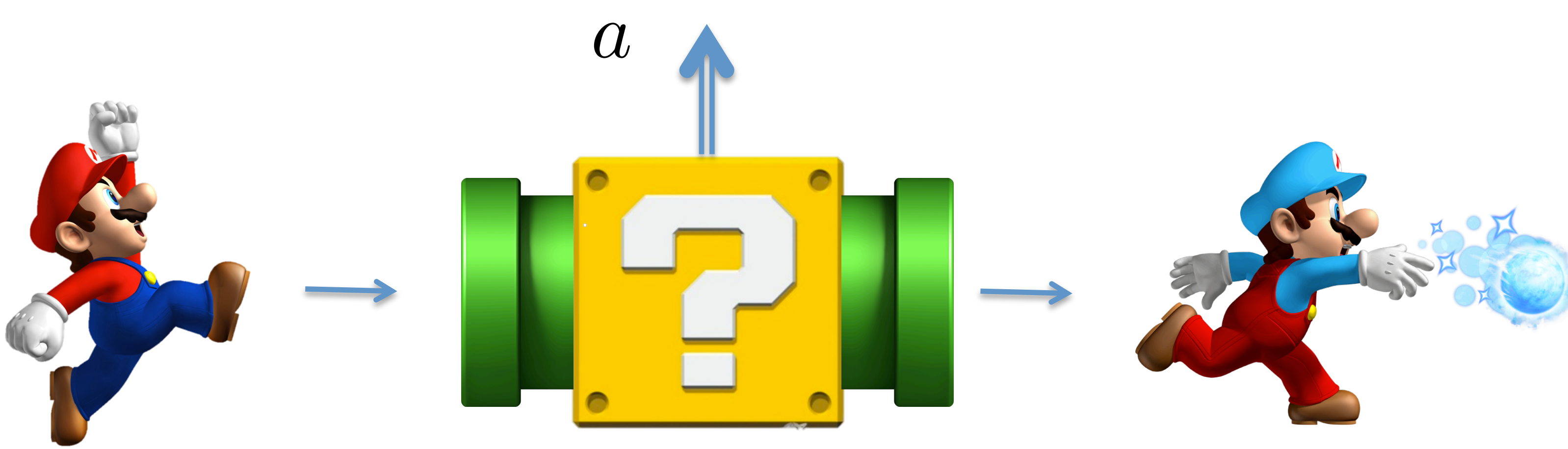
minimum-error subchannel discrimination

Which Λ_a *?*

transformation/
evolution

$$\{\Lambda_a\}_a$$

(instrument)



initial state

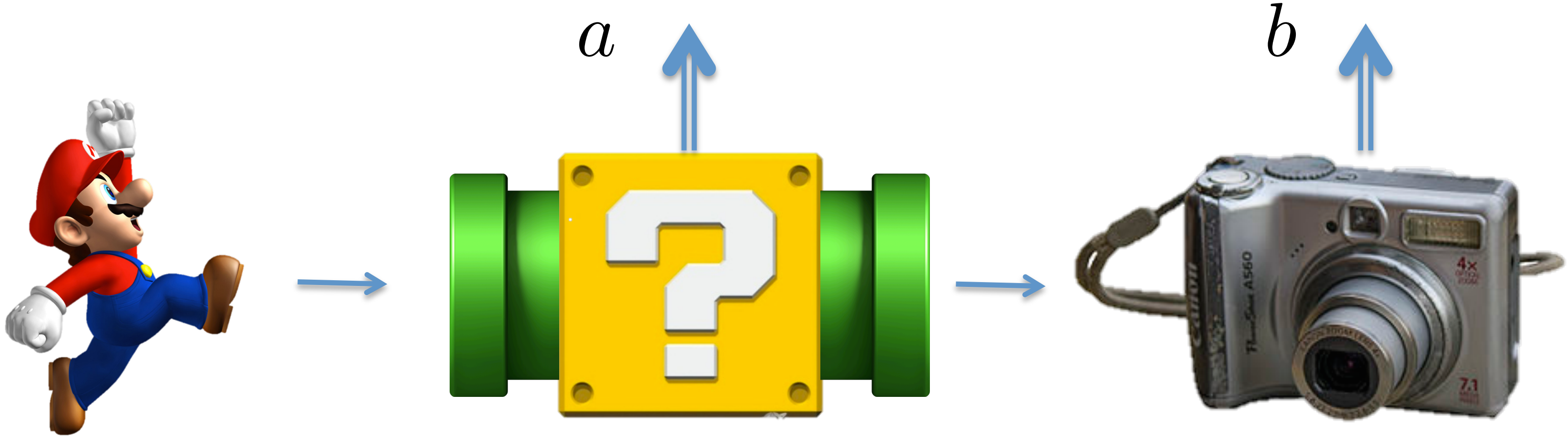
$\hat{\rho}$

transformation/
evolution

$\{\Lambda_a\}_a$

(instrument)

$$p(b, a|\rho) = \mathrm{Tr}(Q_b \Lambda_a[\rho])$$



initial state	transformation/ evolution	measurement
$\hat{\rho}$	$\{\Lambda_a\}_a$	$\{Q_b\}_b$
	(instrument)	(POVM)

Want to optimize the

probability of guessing correctly

$$p_{\mathrm{corr}}(\{\Lambda_a\}_a, \{Q_b\}_b, \hat{\rho}) = \sum_{a,b} p(b, a|\hat{\rho})\delta_{a,b}$$

$$= \sum_a \mathrm{Tr}(Q_a \Lambda_a[\hat{\rho}])$$

same index

Optimal probability of guessing with given input

$$p_{\rm corr}(\{\Lambda_a\}_a,\rho) := \max_{\{Q_b\}_b} p_{\rm corr}(\{\Lambda_a\}_a,\{Q_b\}_b,\rho)$$

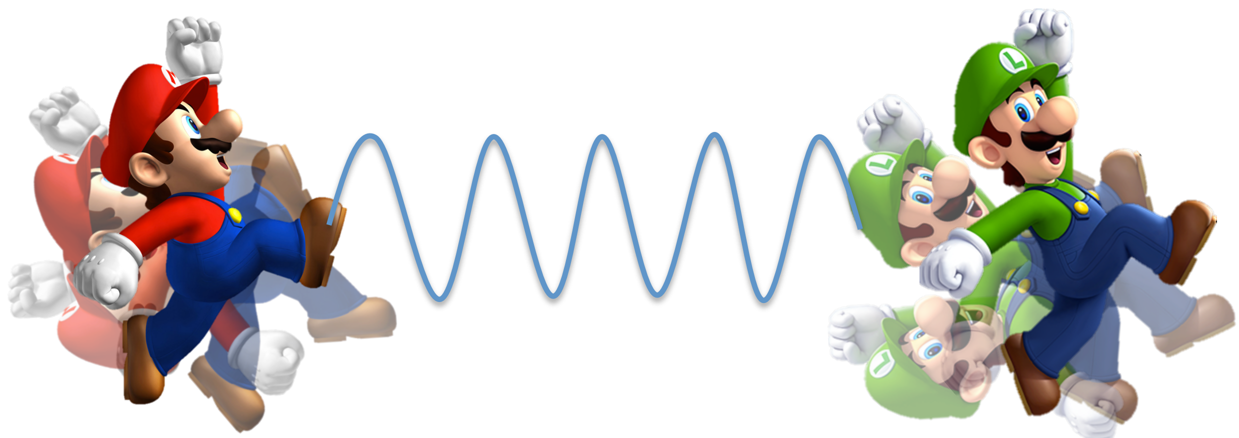
Optimal probability of guessing with optimal input

No **E**ntanglement

$$p^{\rm NE}_{\rm corr}(\{\Lambda_a\}_a) := \max_{\rho} p_{\rm corr}(\{\Lambda_a\}_a,\rho)$$

probe
(a.k.a. Bob,
a.k.a. Mario)

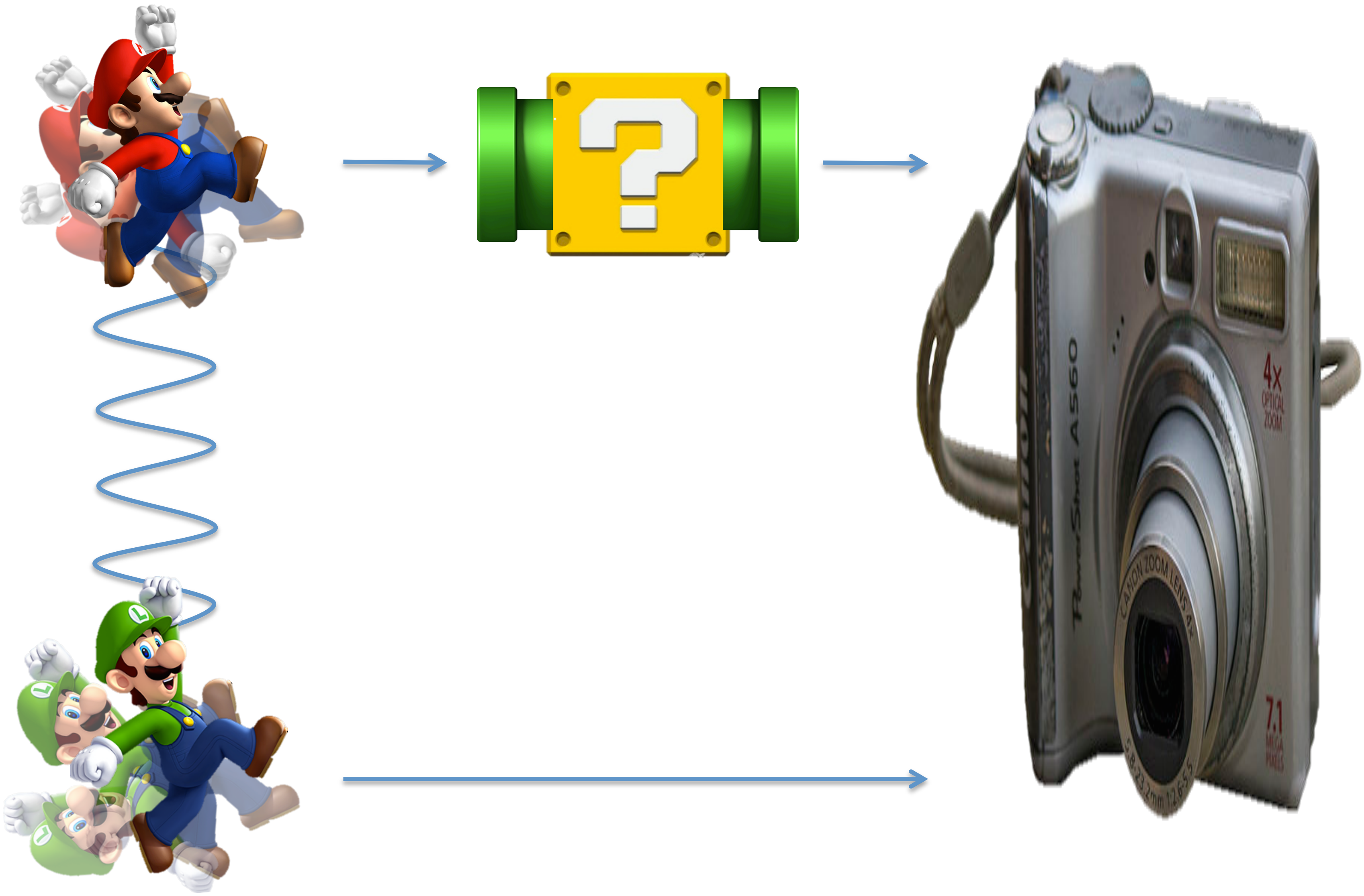
ancilla
(a.k.a. **A**lice,
a.k.a. Luigi)

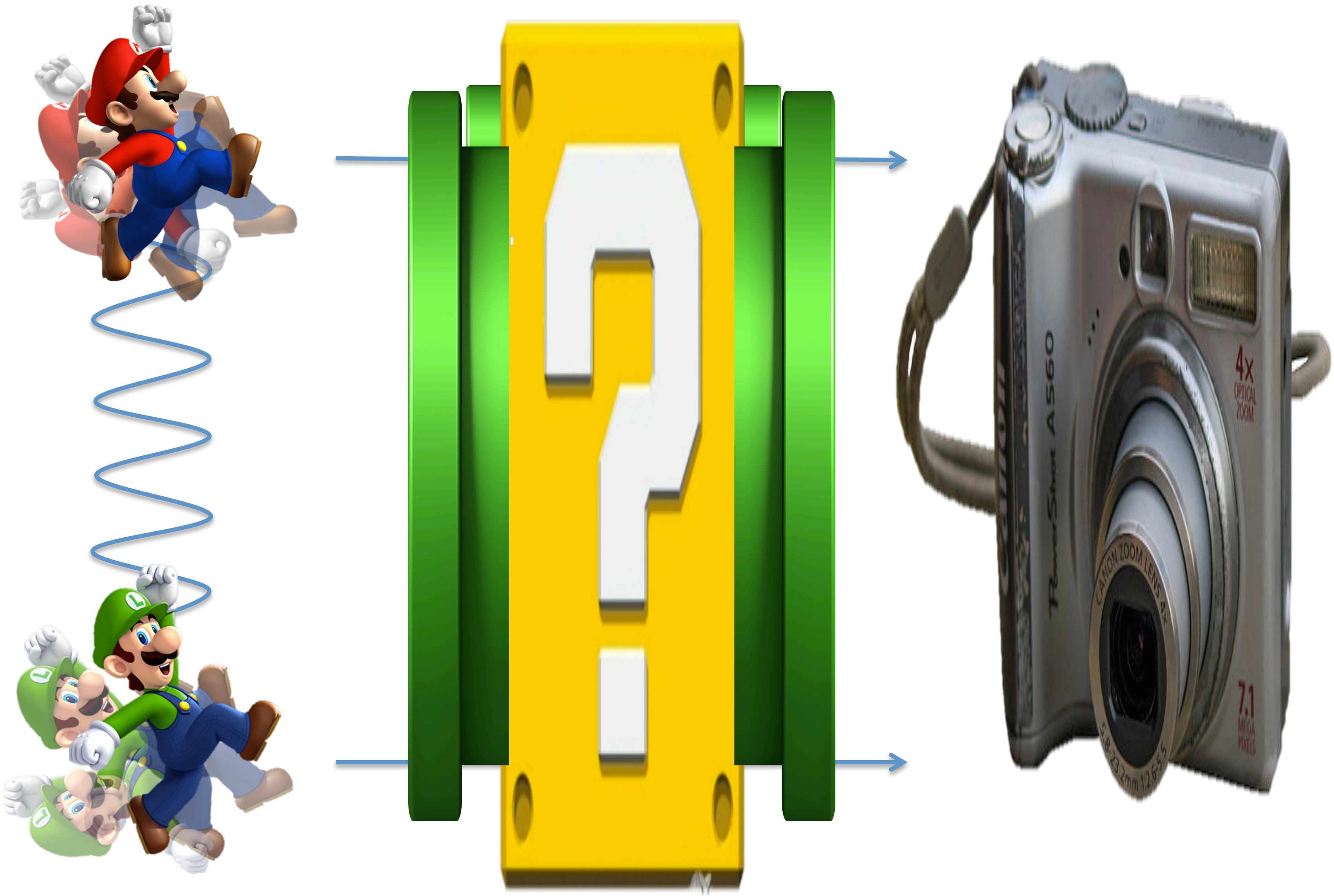


entangled probe and ancilla

$$\hat{\rho}_{AB}^{\text{ent}} \neq \underbrace{\sum_{\lambda} p(\lambda)\hat{\sigma}_A(\lambda) \otimes \hat{\sigma}_B(\lambda)}_{\hat{\sigma}_{AB}^{\text{sep}}}$$

$\hat{\sigma}_{AB}^{\text{sep}}$ separable/unentangled


A560
4x
OPTICAL ZOOM
7.1
MEGA PIXELS


A560
4x
OPTICAL ZOOM
7.1
MEGA PIXELS

Optimal probability of guessing with optimal input, including the possibility of using entanglement

Entanglement

$$p^{\mathrm{E}}_{\mathrm{corr}}(\{\Lambda_a\}_a) := \max_{\text{ancilla } A} p^{\mathrm{NE}}_{\mathrm{corr}}(\{\Lambda_a \otimes \mathrm{id}_A\}_a)$$

ancilla does not evolve

There are evolutions that are **better** distinguished by the use of entanglement

$$p_{\mathrm{corr}}^{\mathrm{E}}(\{\Lambda_a\}_a) > p_{\mathrm{corr}}^{\mathrm{NE}}(\{\Lambda_a\}_a)$$

[Kitaev, Russ. Math. Surv. ’97; Paulsen, *Completely bounded maps and opeator algebras,* ‘02; many others…]

There are evolutions that are **better** distinguished by the use of entanglement

$$p_{\mathrm{corr}}^{\mathrm{E}}(\{\Lambda_a\}_a) > p_{\mathrm{corr}}^{\mathrm{NE}}(\{\Lambda_a\}_a)$$

[Kitaev, Russ. Math. Surv. ’97; Paulsen, *Completely bounded maps and opeator algebras,* ‘02; many others…]

REMARK:
The classical correlations of *unentangled states are useless*!

There are evolutions that are **better** distinguished by the use of entanglement

MOREOVER

For **any** probe-ancilla entangled state, there is a choice of evolutions that are better distinguished using that entangled state

$$p_{\rm corr}(\{\Lambda_a(\hat{\rho}^{\rm ent}_{AB})\}_a, \hat{\rho}^{\rm ent}_{AB}) > p^{\rm NE}_{\rm corr}(\{\Lambda_a(\hat{\rho}^{\rm ent}_{AB})\}_a)$$

[P. and Watrous, PRL '09]

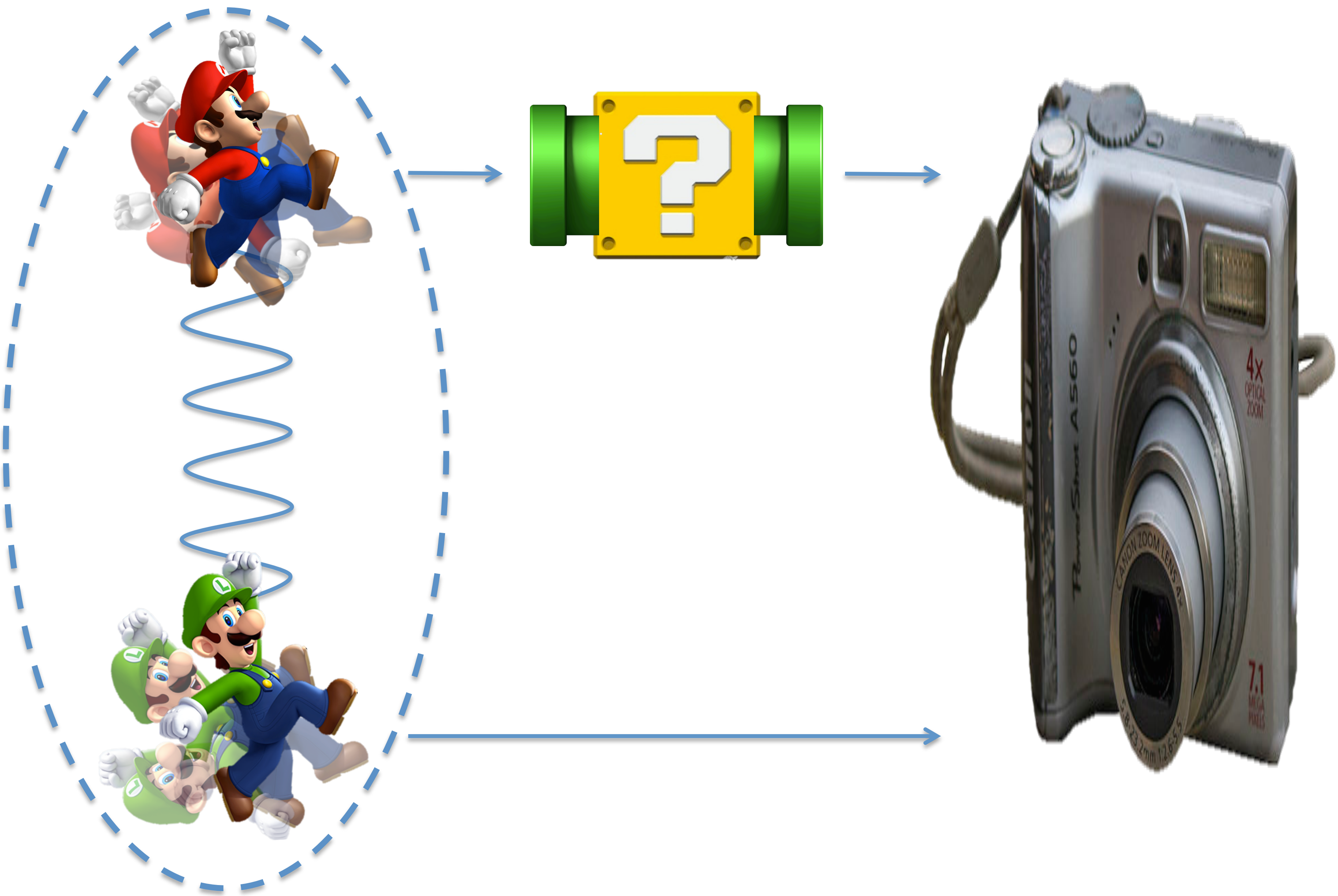
There are evolutions that are **better** distinguished by the use of entanglement

MOREOVER

For **any** probe-ancilla entangled state, there is a choice of evolutions that are better distinguished using that entangled state

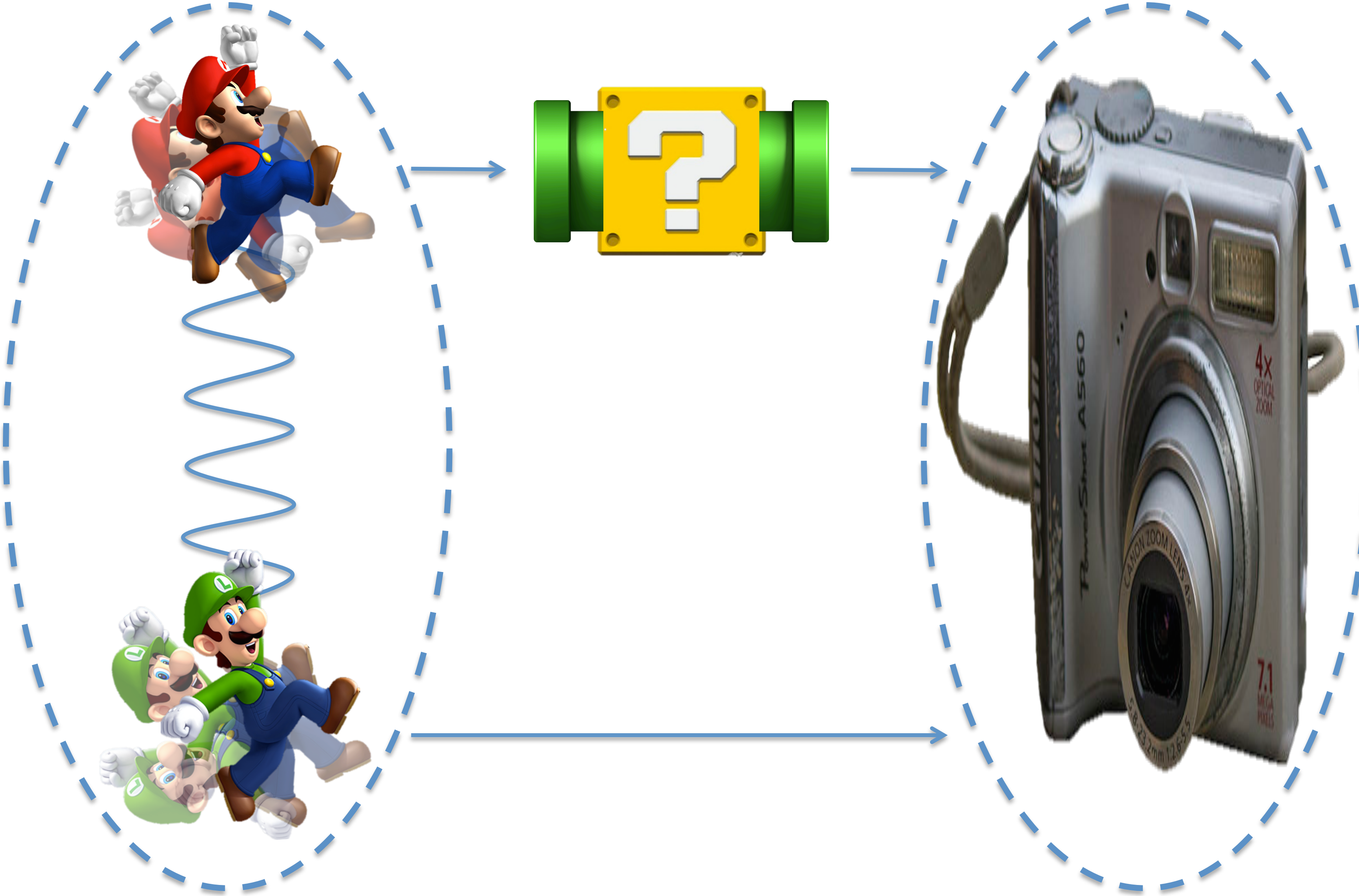
Every entangled state is useful for (sub)channel discrimination

resource!

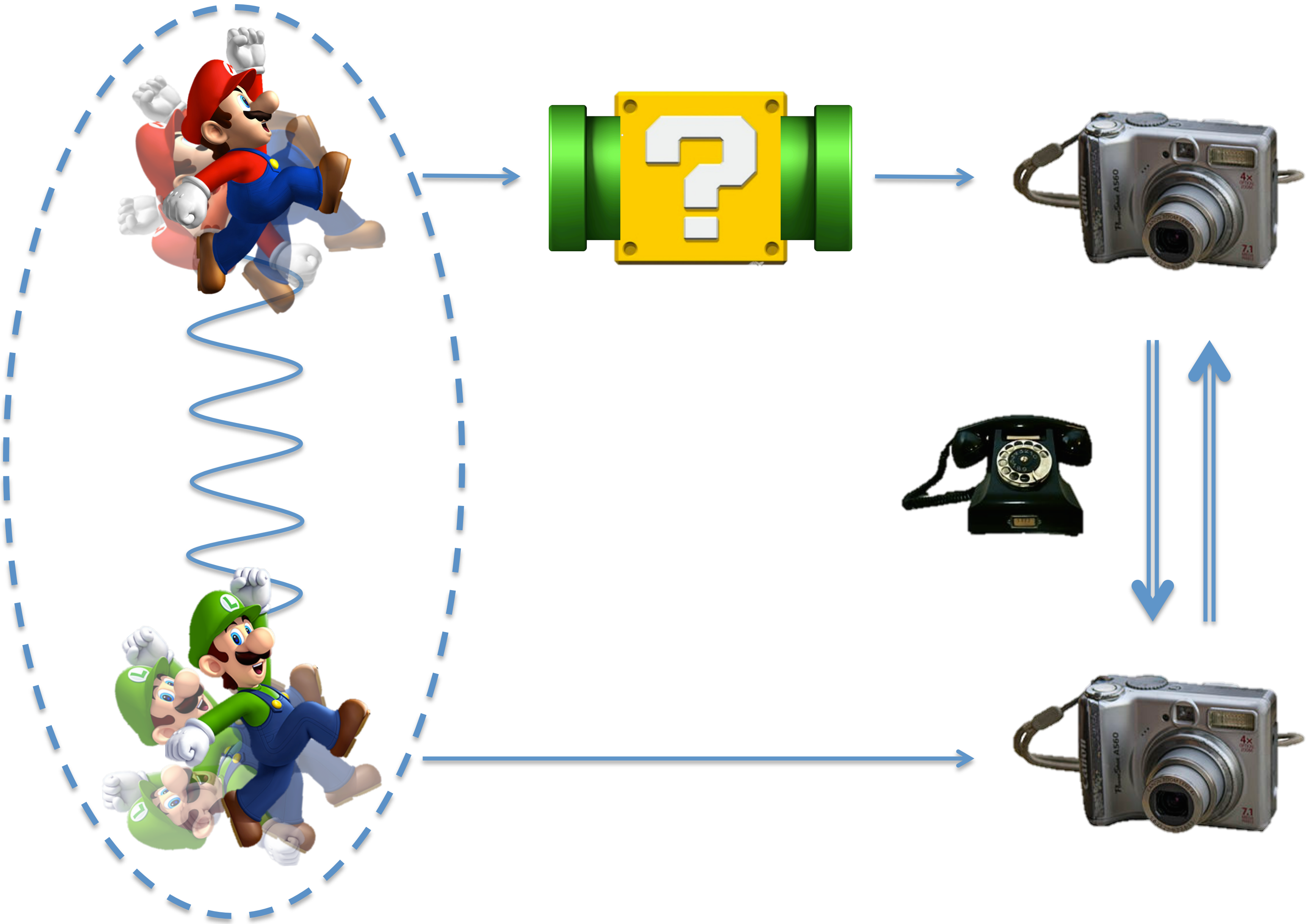


the only resource?

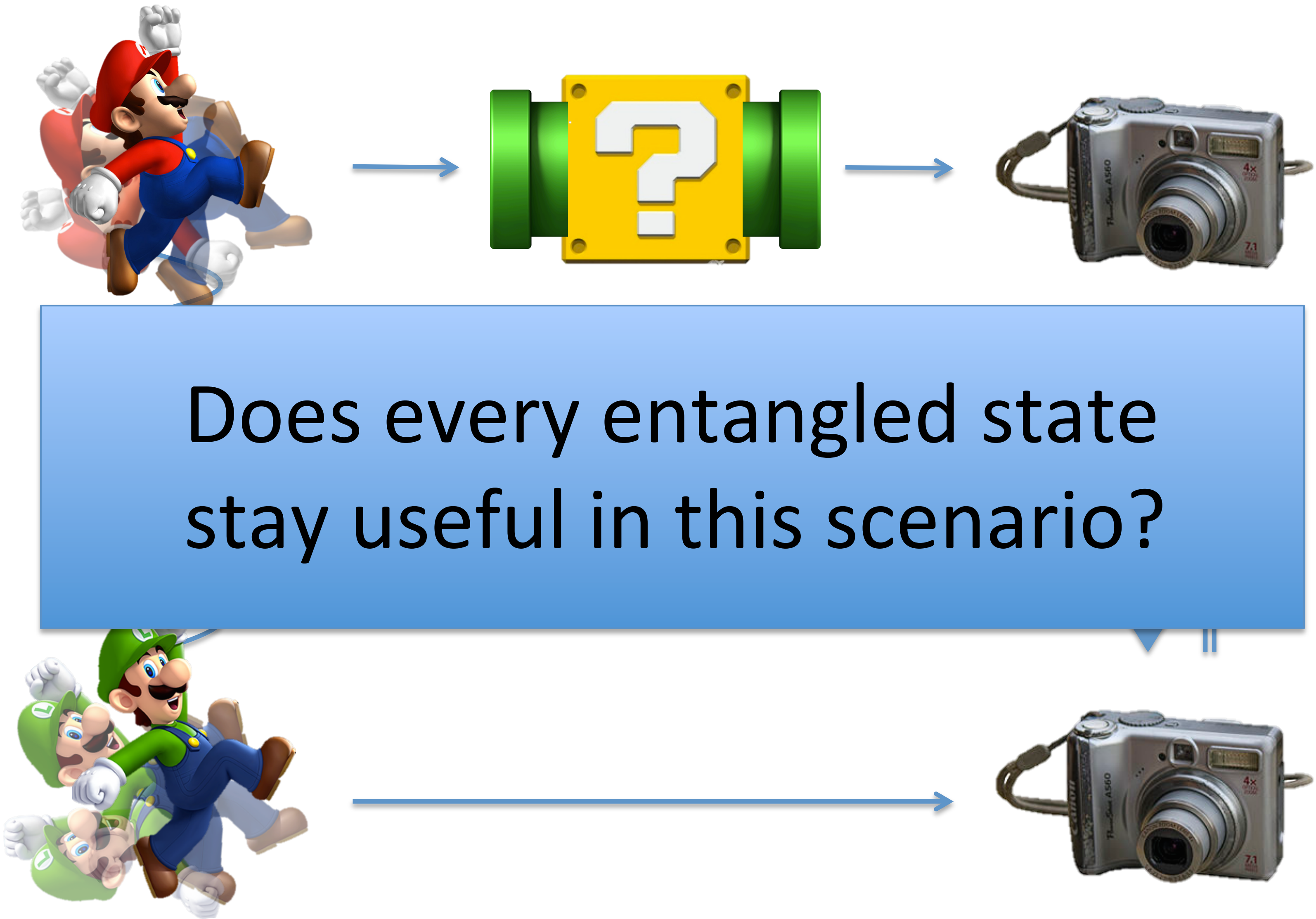
resource?



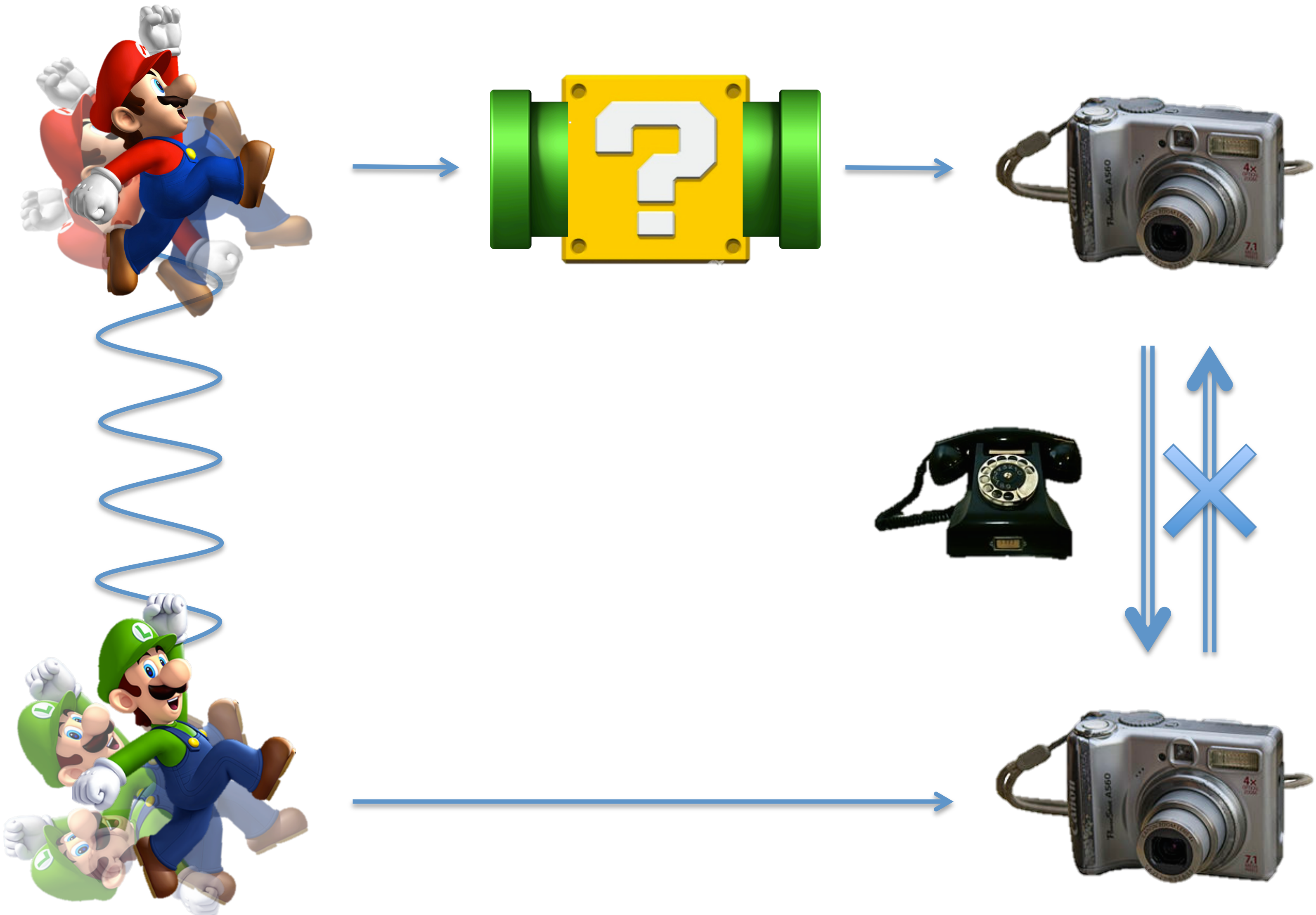
RESOURCE!!!

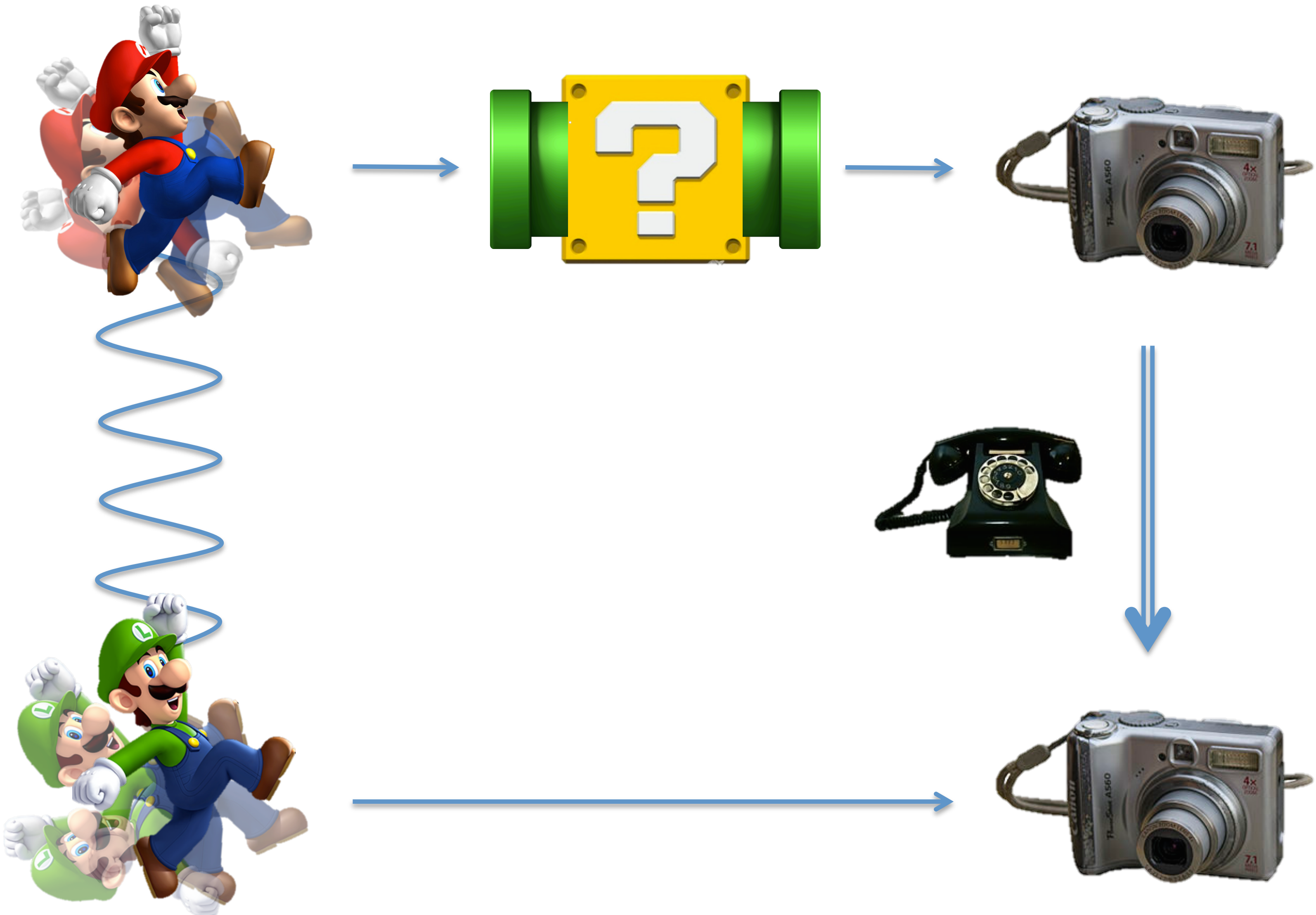


[Matthews, P. and Watrous, PRA '10]



Does every entangled state stay useful in this scenario?



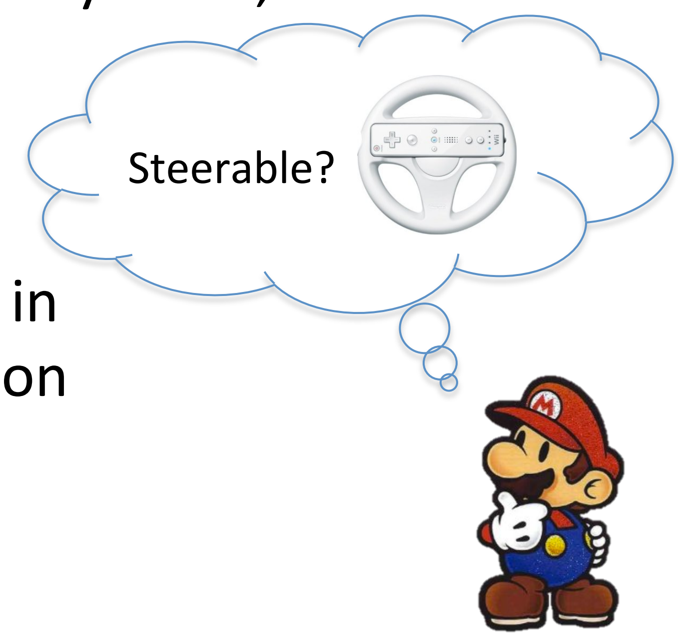


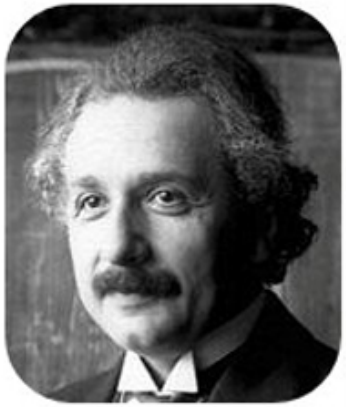
MAIN RESULTS

If measurements are restricted to one-way LOCC, **only steerable states can remain useful**

If measurements are restricted to one-way LOCC, **all steerable states do remain useful!**

The usefulness of a probe-ancilla state in one-way-LOCC subchannel discrimination **quantifies** its **steerability**





Einstein Podolsky Rosen

[see above, Phys. Rev. ‘35]

Schroedinger

[Schroedinger, Proc. Camb. Phil. Soc. ’35, ’36]

STEERING

Alice controls the **conditional** states of Bob through her choice of measurements

POVM element

$$\rho_{a|x}^{B} = \mathrm{Tr}_A(M_{a|x}^{A}\rho^{AB})$$

outcome of measurement

choice of measurement

EXAMPLE OF STEERING

$$\hat{\rho}^{AB} = |\psi^-\rangle\langle\psi^-|^{AB} \qquad |\psi^-\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

$$M^A_{a|0} \in \{|0\rangle\langle 0|, |1\rangle\langle 1|\}$$

$$\rho^B_{a|0} \in \left\{ \frac{1}{2}|1\rangle\langle 1|, \frac{1}{2}|0\rangle\langle 0| \right\}$$

$$M^A_{a|1} \in \{|+\rangle\langle +|, |-\rangle\langle -|\}$$

$$\rho^B_{a|1} \in \left\{ \frac{1}{2}|-\rangle\langle -|, \frac{1}{2}|+\rangle\langle +| \right\}$$

ensembles

assemblage

When is steering *really* quantum?
("spooky action at a distance")

Can we or can we not imagine that
B was in some pre-existing
local hidden state?

Local hidden state model

label of hidden state

$$\{p(\lambda), \hat{\sigma}^B(\lambda)\}$$

probability distribution on hidden states

hidden state

conditional probability

$$\rho^B_{a|x} = \sum_\lambda p(a|x,\lambda)p(\lambda)\hat{\sigma}(\lambda)$$

[Wiseman, Jones, Doherty, PRL ‘07]

Local hidden state model

label of hidden state

$$\{p(\lambda), \hat{\sigma}^B(\lambda)\}$$

probability distribution on hidden states

hidden state

Un**S**teerable (assemblage)

$$\rho_{a|x}^{B,\mathrm{US}} = \sum_{\lambda} p(a|x,\lambda)p(\lambda)\hat{\sigma}(\lambda)$$

$$= \sum_{\substack{\lambda: \\ \text{det.} \\ \text{strat.}}} D(a|x,\lambda)p'(\lambda)\hat{\sigma}'(\lambda)$$

deterministic strategy/response

[Wiseman, Jones, Doherty, PRL '07]

Not unsteerable = steerable

A bipartite state is steerable if it can generate steerable assemblages via local measurements; otherwise unsteerable

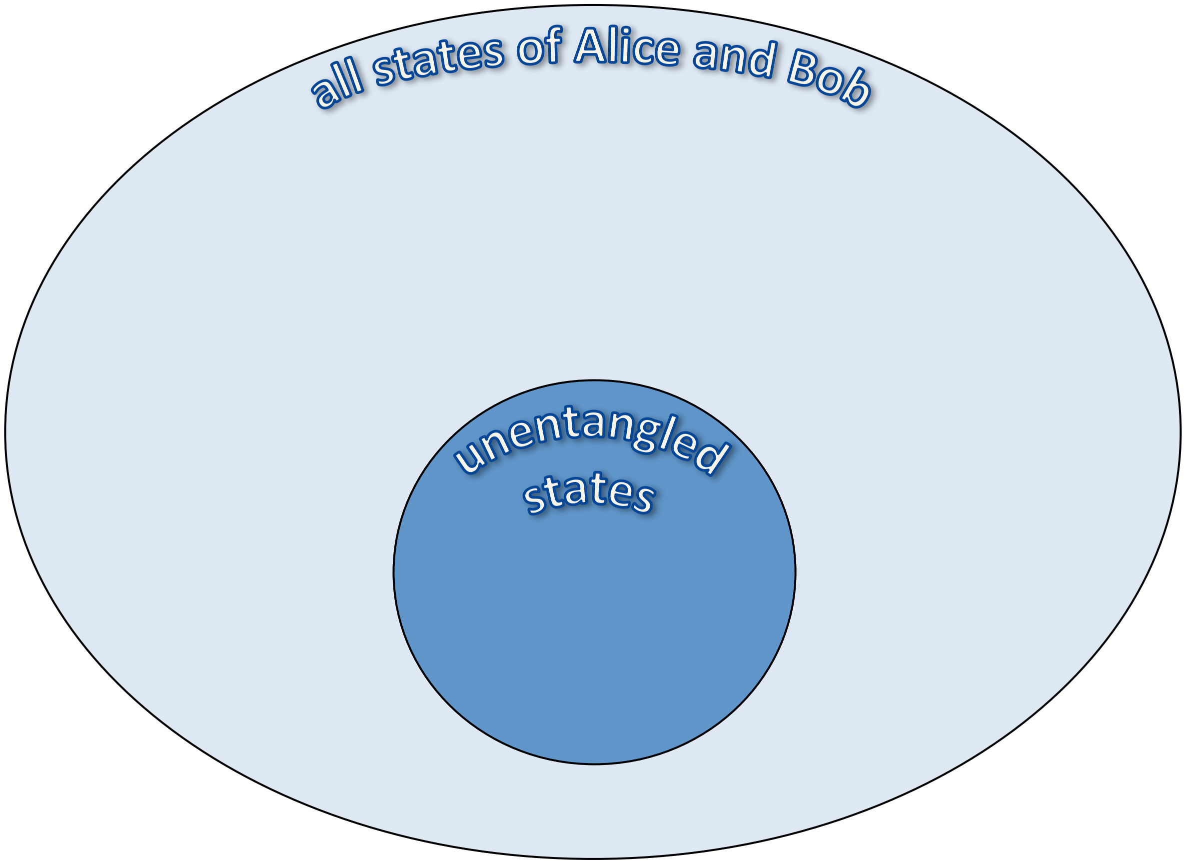
All unentangled states are unsteerable, and all unsteerable assemblages can be seen as originating from some unentangled state:

steering $\Rightarrow$ **entanglement**

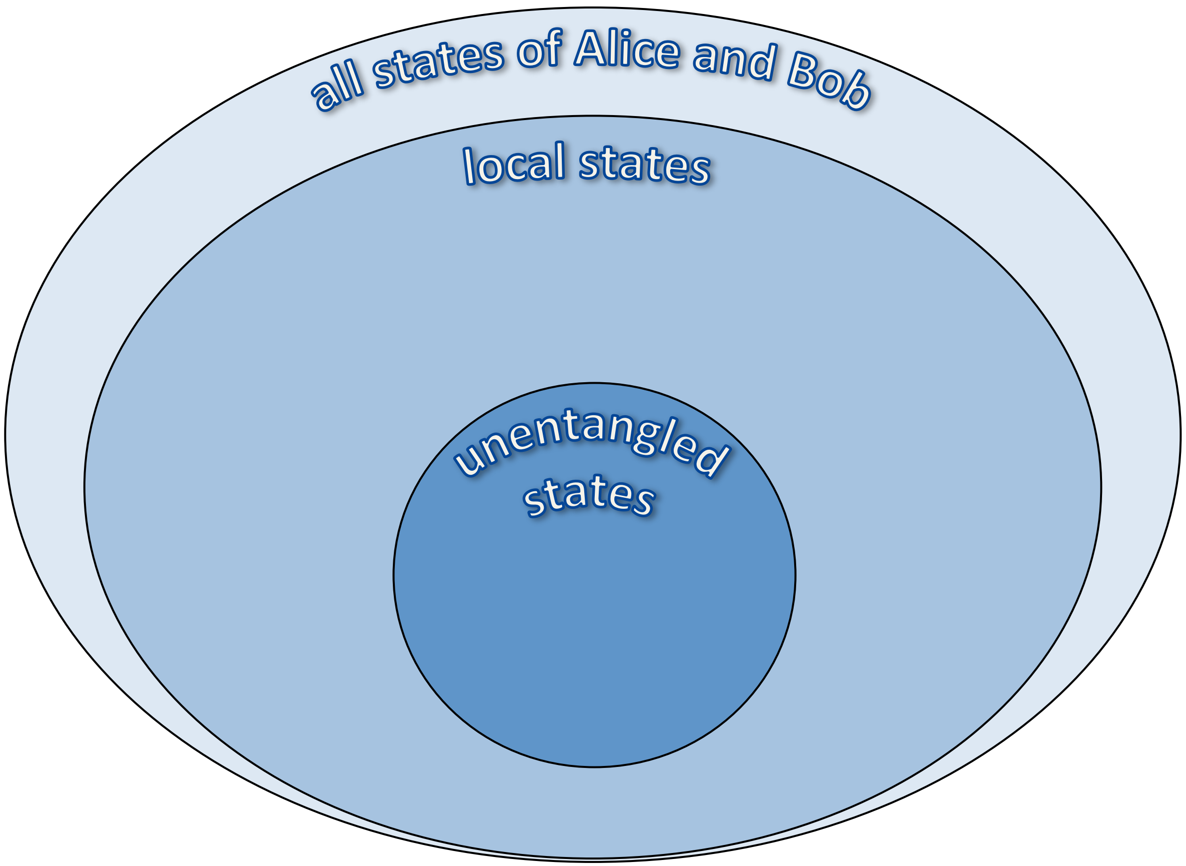
Also some entangled states are unsteerable!!!

steering $\not\Leftarrow$ **entanglement**

[Wiseman, Jones, Doherty, PRL ‘07]



A hierarchy for
bipartite correlations



A hierarchy for
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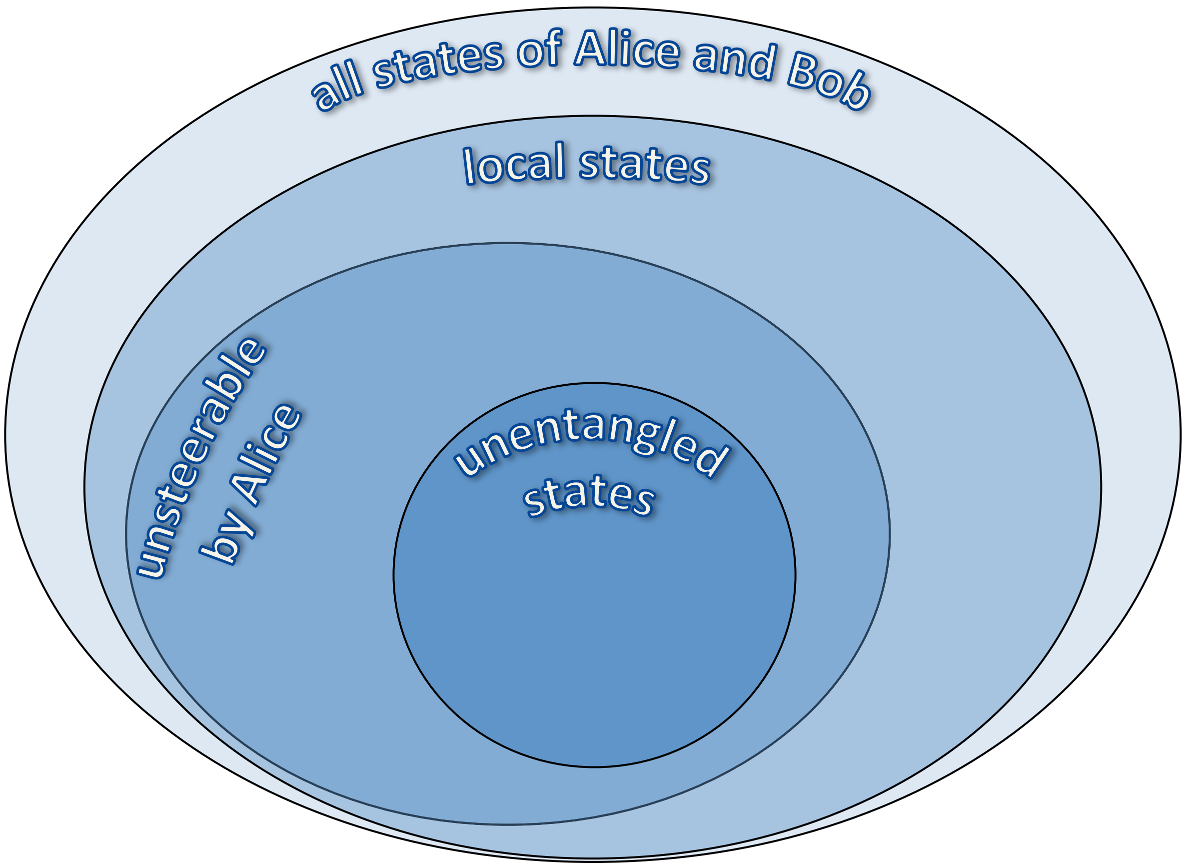
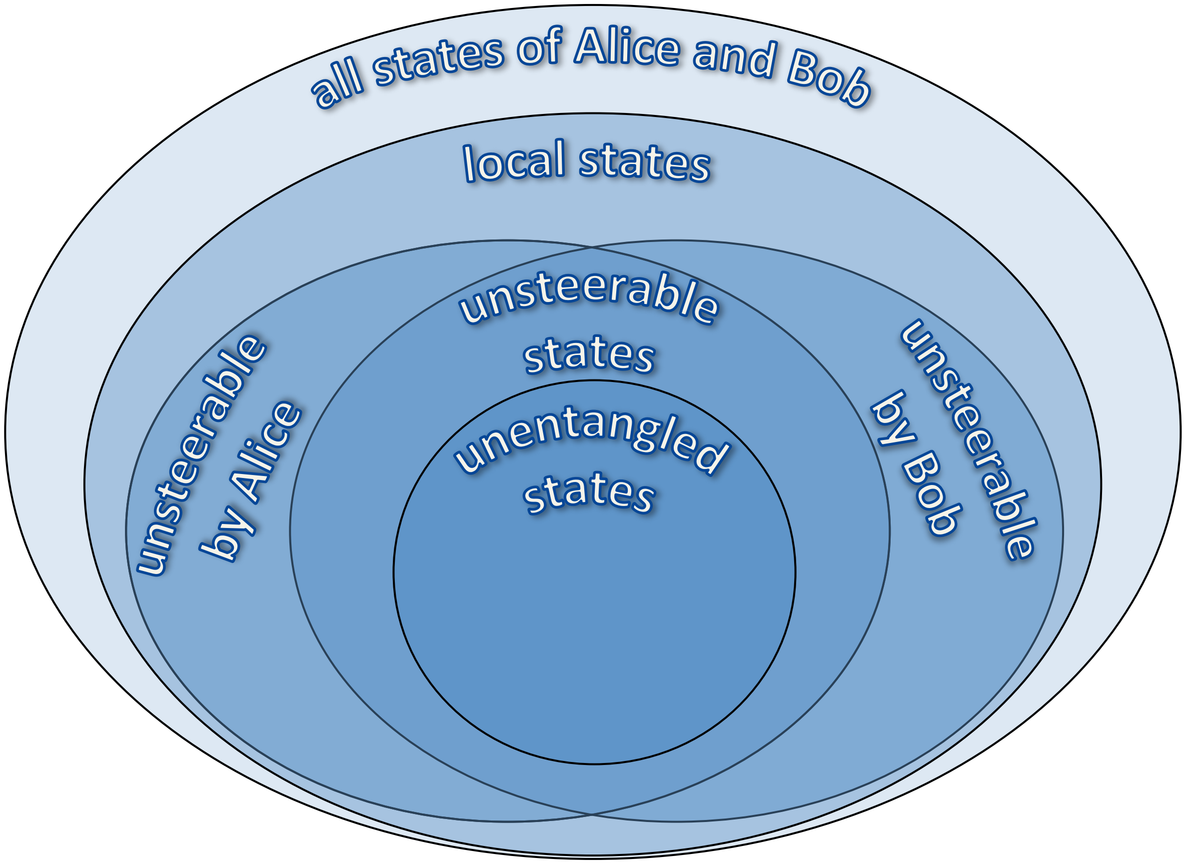
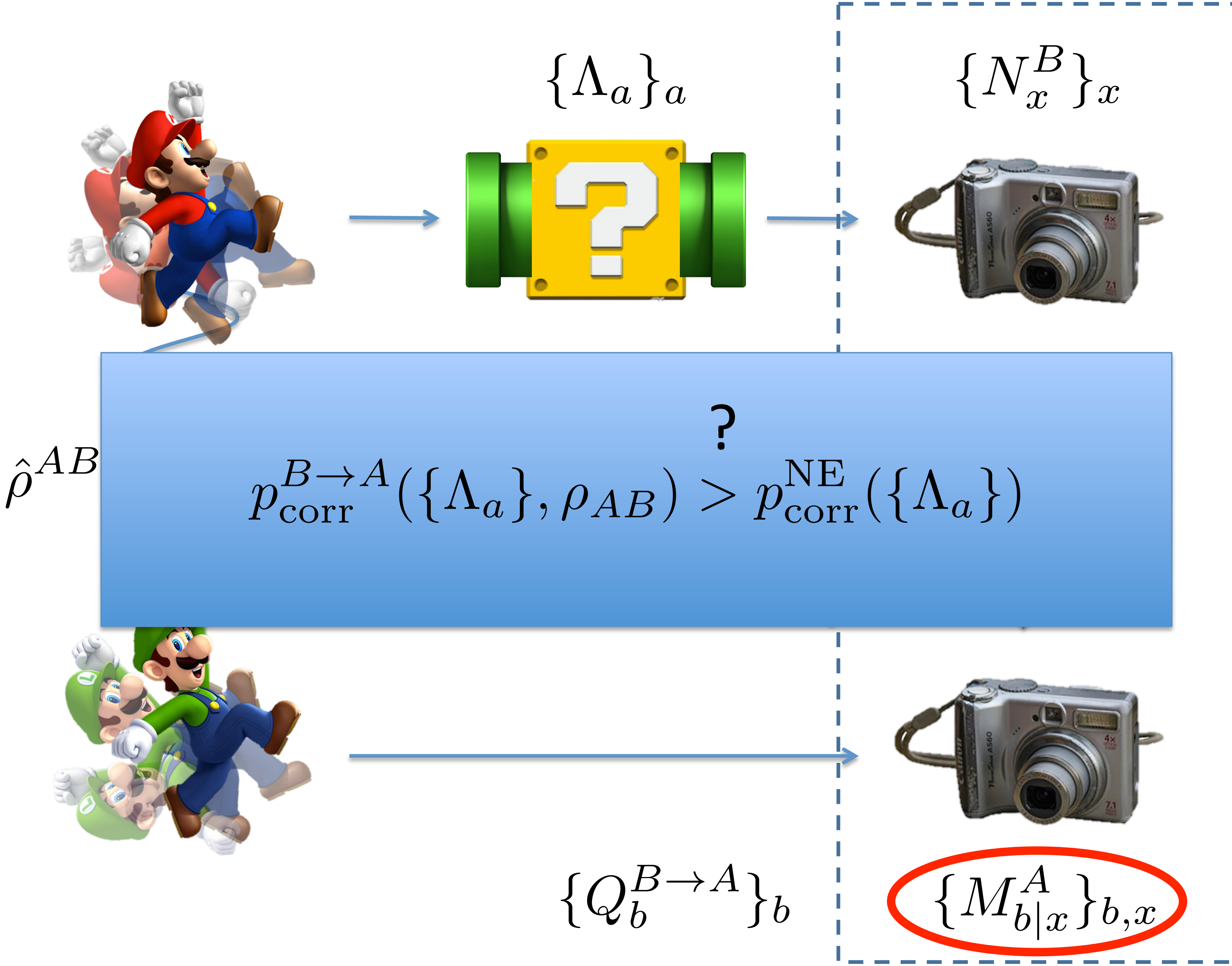


A hierarchy for
bipartite correlations



A hierarchy for
bipartite correlations

The border we characterize operationally



$\{\Lambda_a\}_a$
$\{N_x^B\}_x$
$\hat{\rho}^{AB}$
?
$p_{\text{corr}}^{B\to A}(\{\Lambda_a\}, \rho_{AB}) > p_{\text{corr}}^{\text{NE}}(\{\Lambda_a\})$
$\{Q_b^{B\to A}\}_b$
$\{M_{b|x}^A\}_{b,x}$

In order to have

$$p_{\text{corr}}^{B \to A}(\{\Lambda_a\}, \rho_{AB}) > p_{\text{corr}}^{\text{NE}}(\{\Lambda_a\})$$

it must be that $\{M_{b|x}^{A}\}_{b,x}$ creates steerable assemblage

(otherwise some separable state would have performed as well, and no better than w/o correlations)

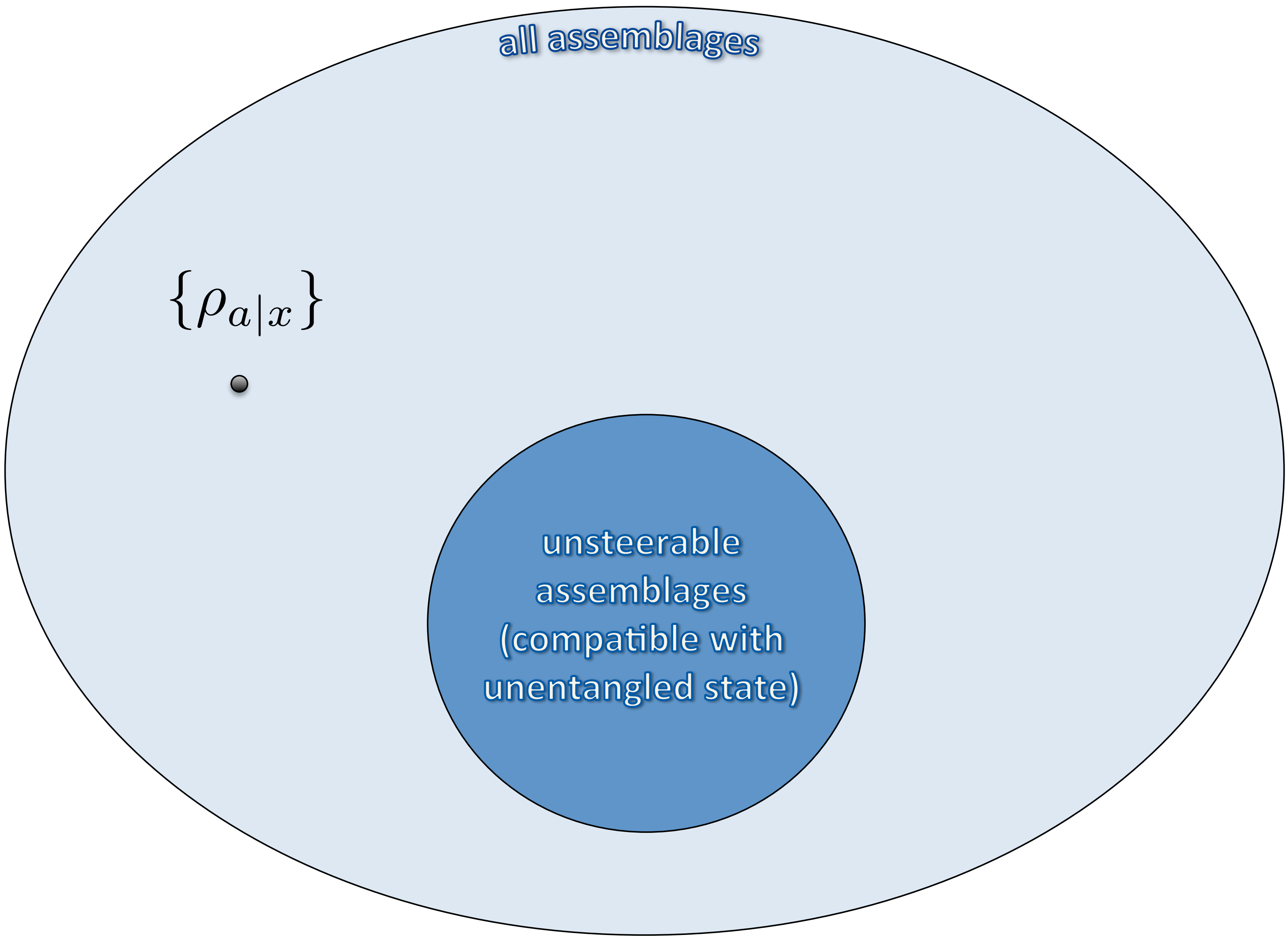
Only steerable states can be useful under the one-way LOCC assumption for measurements
[also entangled states are useless, if unsteerable!!!]

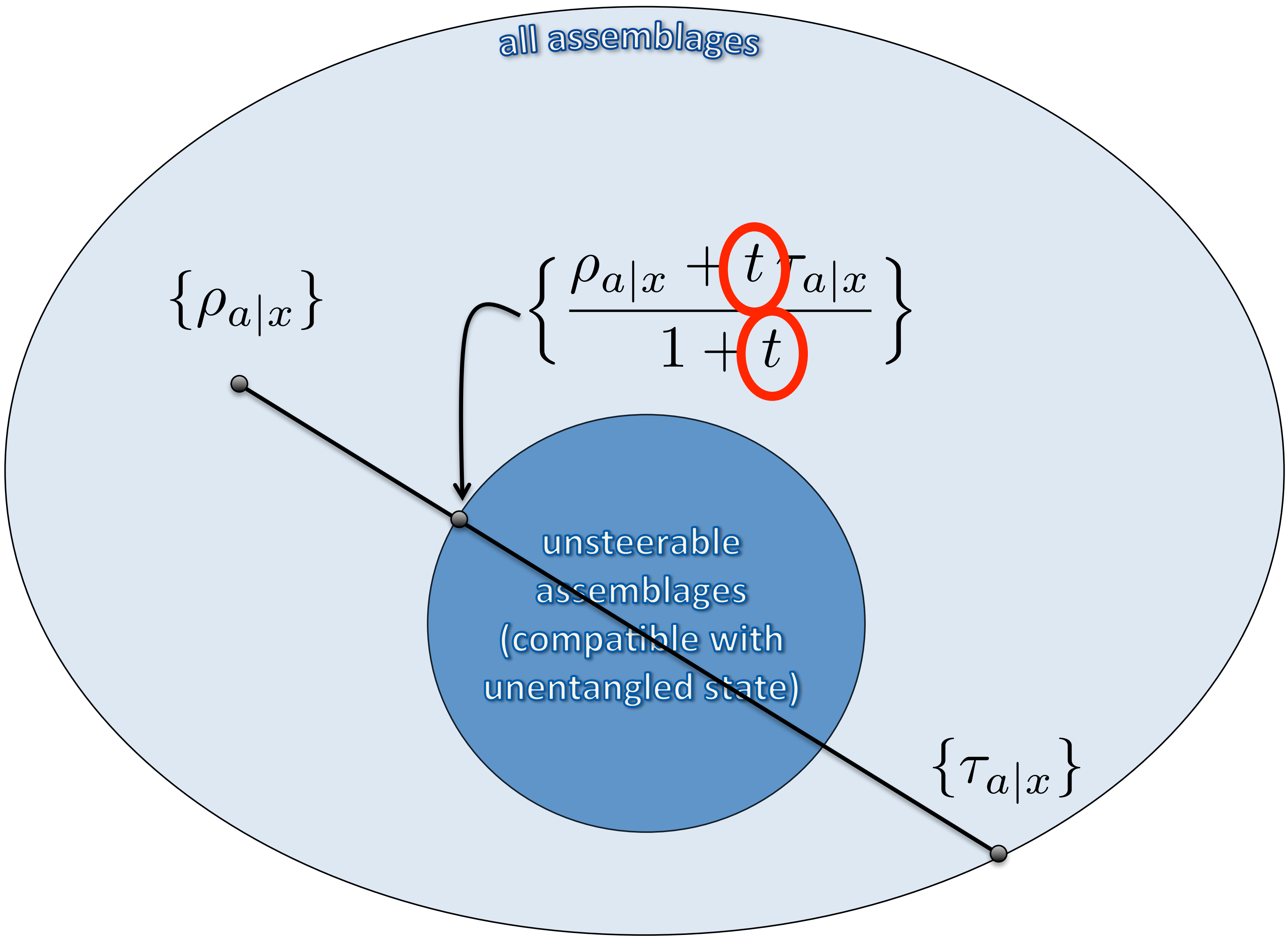
We prove that all steerable states **do stay useful!!!**

If the state is steerable, consider any choice of $\{M^A_{b|x}\}_{b,x}$ that generates a steerable assemblage $\{\rho^B_{a|x}\}_{a,x}$

The **robustenss of steering** of such an assemblage is:

$$R(\{\rho_{a|x}\}) := \min\left\{ t \geq 0 \,\middle|\, \left\{ \frac{\rho_{a|x} + t\,\tau_{a|x}}{1+t} \right\}_{a,x} \text{ unsteerable}, \; \{\tau_{a|x}\} \text{ an assemblage} \right\}$$


all assemblages
$\{\rho_{a|x}\}$
unsteerable assemblages (compatible with unentangled state)



all assemblages
$\{\rho_{a|x}\}$
$\left\{ \frac{\rho_{a|x} + t\,\tau_{a|x}}{1+t} \right\}$
unsteerable assemblages (compatible with unentangled state)
$\{\tau_{a|x}\}$

We define the **steering robustness of the state** as

$$R^{A\to B}_{\mathrm{steer}}(\rho_{AB}) := \sup_{\{M^A_{a|x}\}_{a,x}} R(\{\rho^B_{a|x}\}_{a,x})$$

We prove

$$\boxed{\sup_{\{\Lambda_a\}_a} \frac{p^{B\to A}_{\mathrm{corr}}(\{\Lambda_a\}_a, \rho_{AB})}{p^{\mathrm{NE}}_{\mathrm{corr}}(\{\Lambda_a\}_a)} = R^{A\to B}_{\mathrm{steer}}(\rho_{AB}) + 1}$$

The direction

$$\sup_{\{\Lambda_a\}_a} \frac{p_{\mathrm{corr}}^{B\to A}(\{\Lambda_a\}_a, \rho_{AB})}{p_{\mathrm{corr}}^{\mathrm{NE}}(\{\Lambda_a\}_a)} \le R_{\mathrm{steer}}^{A\to B}(\rho_{AB}) + 1$$

is easily proven just by making use of definitions.

That the upper bound can be achieved is proven by constructing suitable subchannel discrimination problems

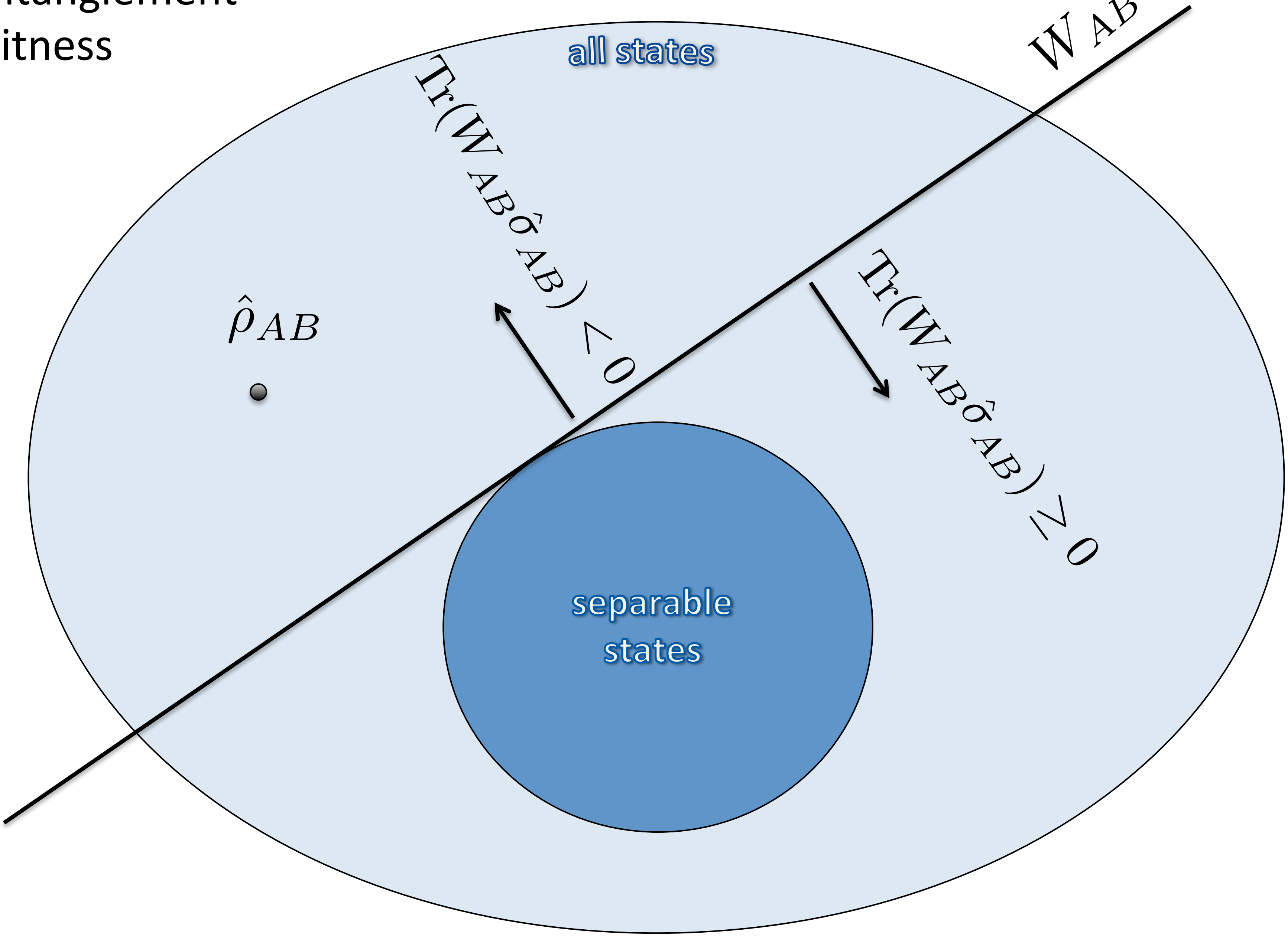
Finding $R(\{\rho_{a|x}\})$ corresponds to a **semidefinite programming (SDP)** optimization problem (whose dual is)

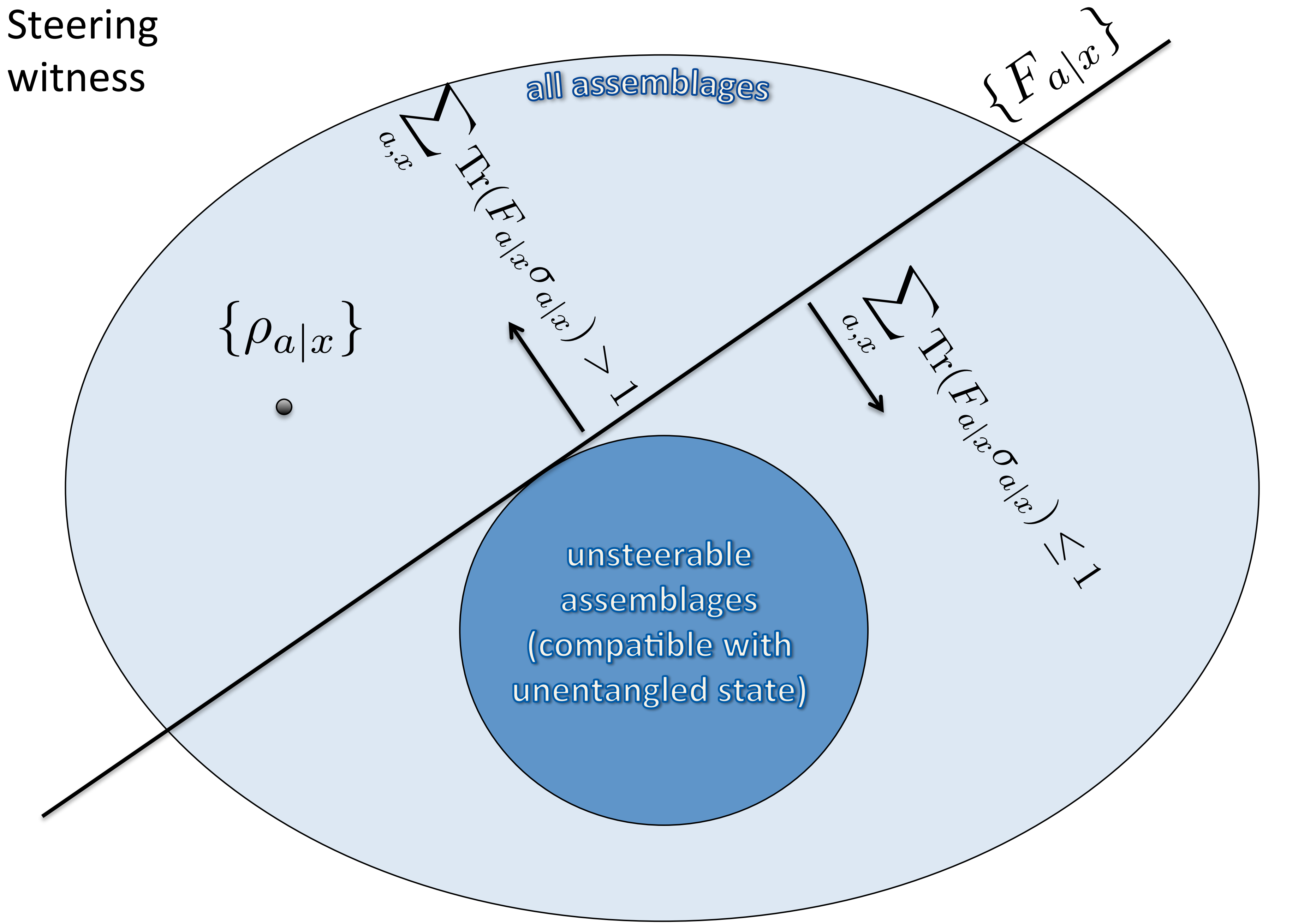
$$\begin{aligned} \text{maximize} \quad & \sum_{a,x} \mathrm{Tr}(F_{a|x}\rho_{a|x}) - 1 \\ \text{subject to} \quad & \sum_{a,x} D(a|x,\lambda) F_{a|x} \leq \mathbb{1} \quad \forall \lambda \\ & F_{a|x} \geq 0 \quad \forall a, x \end{aligned}$$

$D(a|x,\lambda)$: deterministic response

λ : identifier of deterministic response

Entanglement witness





Steering witness
all assemblages
$\{F_{a|x}\}$
$\sum_{a,x} \mathrm{Tr}(F_{a|x}\sigma_{a|x}) > 1$
$\{\rho_{a|x}\}$
$\sum_{a,x} \mathrm{Tr}(F_{a|x}\sigma_{a|x}) \leq 1$
unsteerable assemblages (compatible with unentangled state)

Using the information provided by the SDP optimization problem we construct suitable subchannels $\{\Lambda_a\}_a$

- Choose them to be quantum-to-classical

$$\Lambda_a[\tau] \propto \sum_x \mathrm{Tr}(F_{a|x}\tau)|x\rangle\langle x|$$

use normalization to make them **sub**channels of an instrument

from the SDP

orthonormal

- Take care of trace preservation by introducing suitable "dummy" subchannels

Having used the $F_{a|x}$s that give $R(\{\rho_{a|x}\})$, with our construction we find

$$\frac{p_{\text{corr}}^{B\to A}(\{\Lambda_a\}_a, \rho_{AB})}{p_{\text{corr}}^{\text{NE}}(\{\Lambda_a\}_a)} \geq \frac{R(\{\rho_{a|x}\}) + 1}{1 + \frac{2}{\alpha N}}$$

normalization factor (independent of *N*)

number of dummy subchannels (arbitrary)

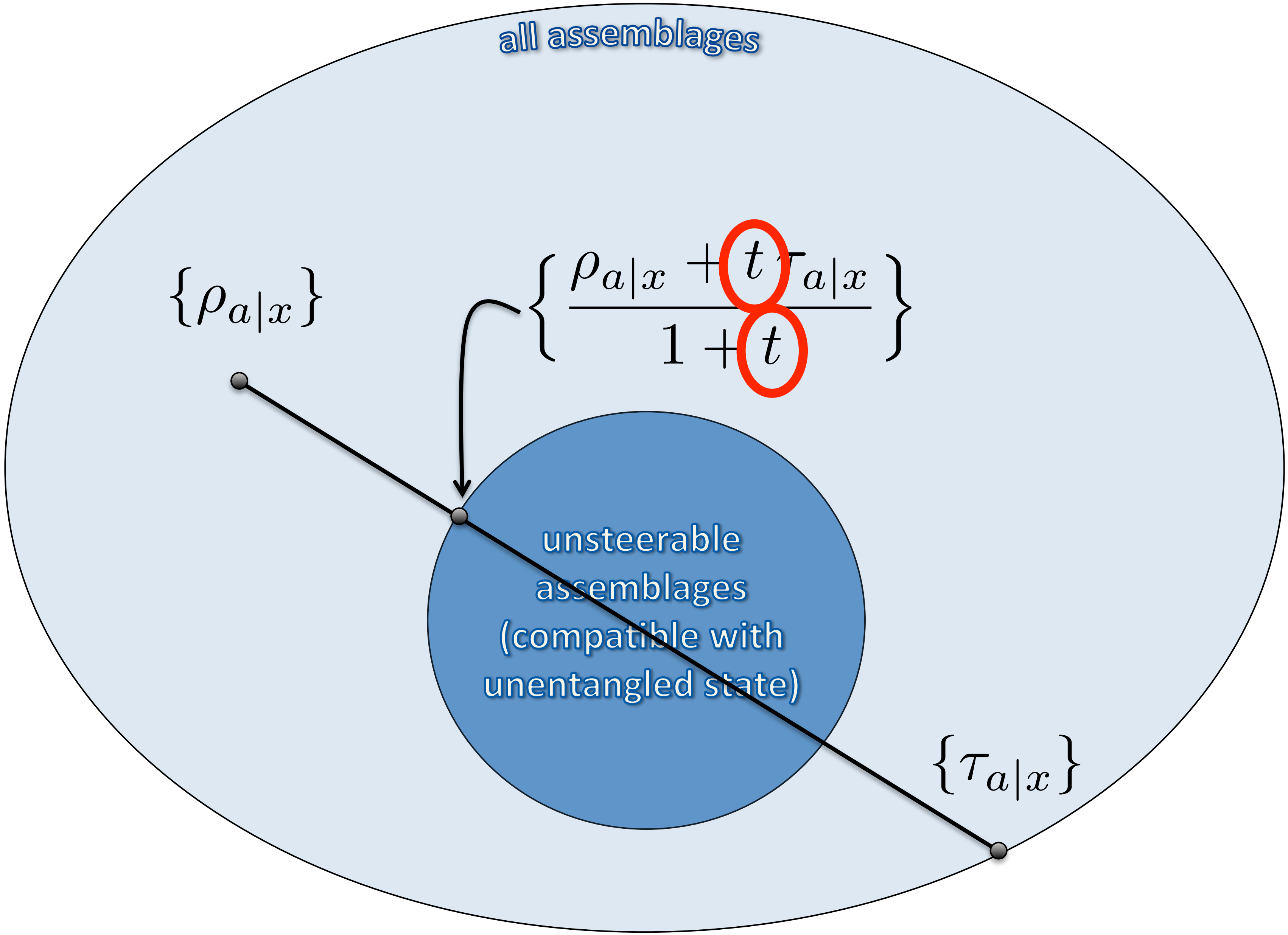
Considering $N \to \infty$ we prove the claim.

REMARK

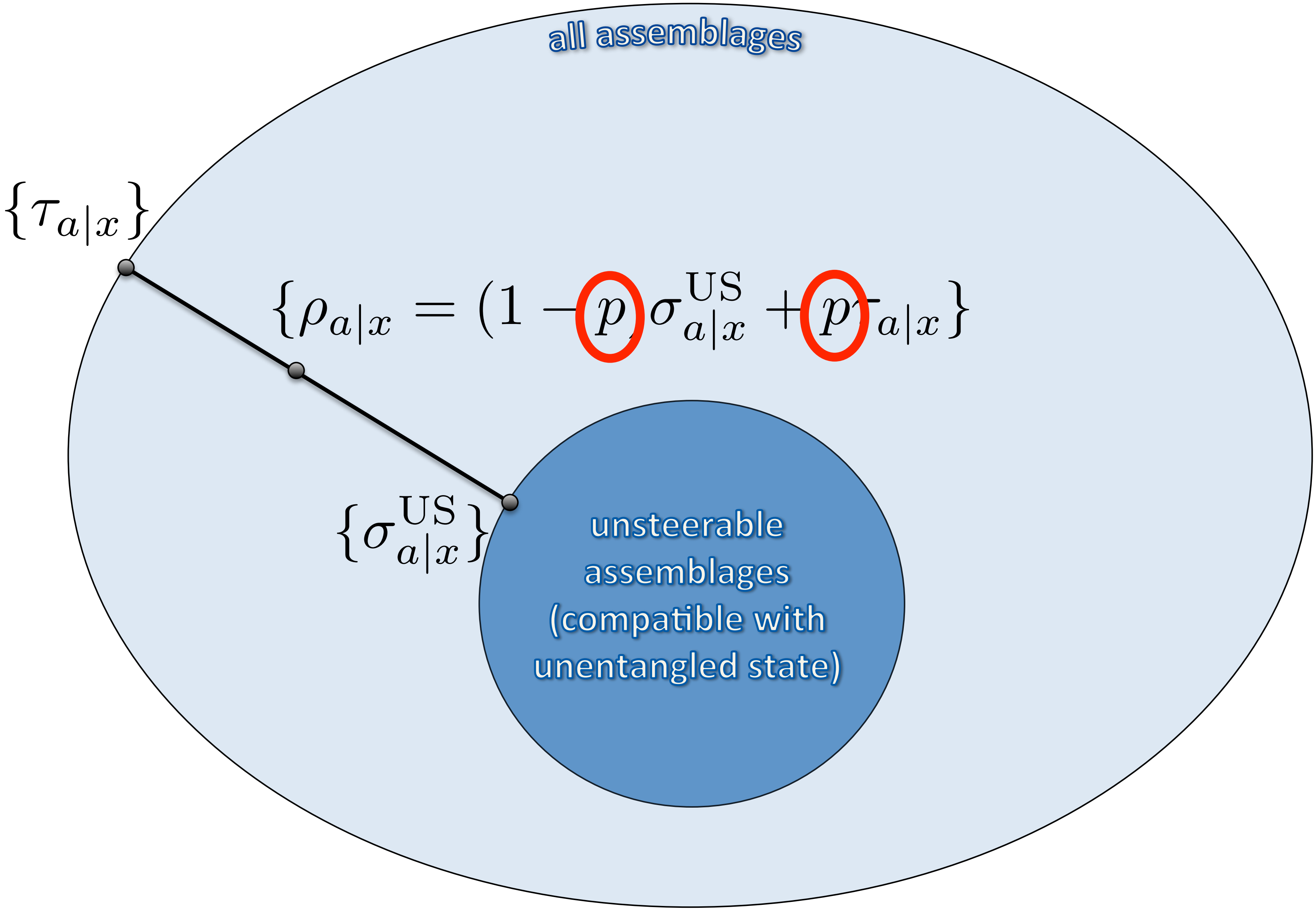
Our SDP approach was also inspired by
[Skrzypczyk, Navascués, and Cavalcanti, PRL ’14]

In their case they use semidefinite programming to compute the so-called *steering weight*

Steering robustness



Steering weight



Conclusions

"All entangled states are special [...]"

All entangled states are useful for (sub)channel discrimination

"[...] but some are more special than others"

Only steerable states can be, and are useful for subchannel discimination under the constraint that the measurements are one-way LOCC

Conclusions

We have introduced the **robustness of steering**:

- it has at least *two* operational interpretations:
  - resilience (of steering) to noise
  - advantage in subchannel discrimination
- computable via SDP for a given assemblage
- it provides semi-device-independent bounds
  to the *robustness of entanglement* [Vidal and Tarrach, PRA '99]
- it scales with the amount of entanglement
- it respects sensible criteria to be considered a resource quantifier [Gallego and Aolita, arXiv:1409.5804]

Some open questions

- Closed formula for the robustness of steering for pure states/maximally entangled states
- Can steering be characterized by considering channel discrimination, rather than *sub*channel discimination?
- Are all entangled states useful for (sub)channel discrimination under general LOCC (Vs one-way LOCC)?
- Can we also characterize non-locality --- besides entanglement and steering --- via (sub)channel discrimination tasks?

CIFAR CANADIAN INSTITUTE FOR ADVANCED RESEARCH

NSERC CRSNG

THANK  YOU!!!

arXiv:1406.0530,
PRL to appear

University of Strathclyde Science

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